

Ratios of Electromagnetic Transition Probabilities in low States of Odd A Rotational Nuclei

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Dedicated to Professor Dr. W. GENTNER on the occasion of his 60th birthday

The systematics of M1 and E2 transition probabilities between low lying states in odd *A* rotational nuclei are reviewed. Some departures are found from strict rotational behaviour in ratios of M1 and E2 transition probabilities in successive transitions.

The systematics of M1 transition probabilities in deformed nuclei in the region $153 \leq A \leq 187$ have been reviewed and discussed in a communication by de BOER and ROGERS in 1963¹. Since then a considerable amount of new information has been accumulated and it seems appropriate now to bring this survey up to date. In particular, information is now available on the transitions $I+1 \rightarrow I$ (The spin values are used as labels for the nuclear states, with *I* being the spin of the ground state^{2,3,4}, whereas the survey of Ref. 1 refers mostly to $I+2 \rightarrow I+1$ transitions. The present review therefore concerns itself primarily with a comparison of the transition probabilities for the two types of transitions indicated.

Data are available now also for the E2 probabilities of the $I+2 \rightarrow I+1$ transitions whereas previously only the E2 probabilities of the $I+1 \rightarrow I$ and $I+2 \rightarrow I$ transitions were known for most of these nuclei⁵.

The relevant measurements and transition probabilities are summarized in Table 1. Ref. 1 quotes *B*(M1) values for the transitions $I+1 \rightarrow I$ in a number of nuclei. For some of these the values quoted in the present review differ somewhat from the values of Ref. 1. These differences are due both to newly accumulated data and to reevaluations and corrections to the old data.

Eu¹⁵³. The value in Ref. 1 was based in part on a lifetime measurement carried out at this laboratory by the center-of-gravity-shift method. A recent

and more reliable measurement⁴ carried out here yielded a different value which is also more consistent with branching ratio measurements.

Tb¹⁵⁹. The value in Ref. 1 was based on lifetime measurements and a rather uncertain extrapolation of the total conversion coefficient. The present value is derived from recent mixing ratio measurements³.

Yb¹⁷³. There is an arithmetical error in the derivation of Ref. 1. The value in this review is derived from the data of Ref. 1 together with more recent data³.

The E2 and M1 reduced transition probabilities within the ground state band are given in terms of the rotational band parameters as:

$$B(E2, I_i \rightarrow I_f) = \frac{5}{16\pi} e^2 Q_0^2 (I_i 2, I 0 | I_f 2, I_f I)^2, \quad (1)$$

$$B(M1; I_i \rightarrow I_f) = \frac{3}{4\pi} \left(\frac{e\hbar}{2Mc} \right)^2 (g_K - g_R)^2 I^2 (I_i 1, I 0 | I_f 1, I_f I)^2. \quad (2)$$

Here Q_0 is the intrinsic quadrupole moment, g_R the gyromagnetic ratio of the rotational motion and g_K the gyromagnetic ratio of the intrinsic motion; the other symbols have their conventional meanings.

For nuclei with $I = \frac{1}{2}$ the expressions for the transition probabilities involve an additional parameter which makes the determination of ratios of transition probabilities somewhat ambiguous. Such nuclei have therefore been excluded from this discussion.

¹ J. DE BOER and J. D. ROGERS, Phys. Letters **3**, 304 [1963].

² D. ASHERY, A. E. BLAUGRUND, R. KALISH, J. S. SOKOLOWSKI, and Z. VAGER, Nucl. Phys. **67**, 385 [1965].

³ D. ASHERY, A. E. BLAUGRUND, and R. KALISH, Nucl. Phys. **76**, 336 [1966].

⁴ D. ASHERY, N. ASSAF, G. GOLDRING, A. SPRINZAK, and Y. WOLFSON, Nucl. Phys. **77**, 650 [1966].

⁵ B. ELBEK, Determination of Nuclear Transition Probabilities by COULOMB Excitation, Ejnar Munksgaards Forlag, Copenhagen 1963.



1 Nu- cleus	2 I_0	3 μ_0	4 $(g_K - g_R)_1^2$	5 $(g_K - g_R)_2^2$	6 $\frac{(g_K - g_R)_1^2}{(g_K - g_R)_2^2}$	7 $g_K(1)$	8 $g_K(2)$	9 $\frac{g_K(1)}{g_K(2)}$	10 $g_R(1)$	11 $g_R(2)$	12 $\frac{g_R(1)}{g_R(2)}$
¹⁵³ Eu	5/2	+ 1.529(2)	0.0328(16) ⁴	0.0345(20) ^{2,4,9}	0.95(7)	+ 0.663(5)	+ 0.659(5)	1.02(1)	0.482(3)	0.478(3)	1.01(1)
¹⁵⁵ Gd	3/2	- 0.242(2)	0.60(14) ³	0.61(9) ^{3,9}	0.98(27)	- 0.471(36)	- 0.473(26)	0.99(9)	0.303(54)	0.307(40)	0.99(22)
¹⁵⁷ Gd	3/2	- 0.3225(10)	0.62 ¹¹	0.706(75) ^{9,11}	0.88	- 0.47	- 0.55(5)	0.85	0.33	0.29(3)	1.14
¹⁵⁹ Tb	3/2	+ 1.52(8)	2.23(10) ³	1.72(30) ¹	1.3(2)	+ 1.51(6)	+ 1.54(7)	0.98(6)	0.117(60)	0.227(80)	0.51(30)
¹⁶³ Dy	5/2	+ 0.635(14)	0.0055(9) ^{4,8}	0.0027(4) ¹¹	2.05(47)	+ 0.233(6)	+ 0.240(6)	0.97(4)	0.307(7)	0.291(7)	1.05(3)
¹⁶⁵ Ho	7/2	+ 4.01(8)	0.92(12) ²	0.682(60) ^{2,11,12}	1.34(21)	+ 1.36(3)	+ 1.33(2)	1.02(3)	0.403(50)	0.504(23)	0.8(1)
¹⁶⁷ Er	7/2	- 0.564(7)	0.46(16) ³	0.137(13) ^{3,7,9,10}	3.4(12)	- 0.312(40)	- 0.244(38)	1.28(20)	0.366(90)	0.127(13)	2.9(8)
¹⁷³ Yb	5/2	- 0.6795(9)	0.87(13) ³	0.497(86) ^{3,9}	1.75(40)	- 0.537(20)	- 0.469(17)	1.14(5)	0.396(50)	0.236(50)	1.68(38)
¹⁷⁵ Lu	7/2	+ 2.230(11)	0.158(15) ³	0.112(16) ^{3,9}	1.41(25)	+ 0.724(5)	+ 0.713(6)	1.02(1)	0.328(15)	0.378(20)	0.87(6)
¹⁷⁹ Hf	9/2	- 0.47(3)	0.244(20) ³	0.202(28) ^{3,12}	1.21(19)	- 0.194(8)	- 0.184(9)	1.05(6)	0.30(2)	0.266(25)	1.13(11)
¹⁸¹ Ta	7/2	+ 2.35(1)	0.271(25) ³	0.204(26) ^{3,9}	1.33(21)	+ 0.788(6)	+ 0.772(7)	1.02(1)	0.267(20)	0.320(23)	0.835(80)
¹⁸⁵ Re	5/2	+ 3.1717	1.55(30) ³	1.34(32) ^{3,12}	1.16(35)	+ 1.61(35)	+ 1.60(4)	1.01(4)	0.365(9)	0.44(10)	0.83(27)
¹⁸⁷ Re	5/2	+ 3.2044	1.44(20) ³	1.33(23) ^{3,12}	1.08(24)	+ 1.62(2)	+ 1.61(3)	1.01(3)	0.423(60)	0.456(70)	0.93(20)

Table 1. Summary of the experimental results for the magnetic properties of ground state rotational bands in odd A nuclei with $K \neq \frac{1}{2}$ in the region $153 \leq A \leq 187$. Column 3: Values from Ref. 6 unless otherwise mentioned. Columns 4, 5: Values for the $I+1 \rightarrow I$ and $I+2 \rightarrow I+1$ rotational transitions respectively. The $B(E2)$ values used in the calculations were taken from Ref. 5.

For nuclei with $I > \frac{1}{2}$ the ratios of reduced transition probabilities are according to (1) and (2) independent of the band parameters. This is a specific prediction of the rotational model which we examine now in detail.

Fig. 1 shows the ratio $B(M1; I+1 \rightarrow I)/B(M1; I+2 \rightarrow I+1)$, divided by the rotational band value given by equation (2). It is clear from the figure that the $B(M1)$ ratios do not follow the rotational pattern at all well and large discrepancies exist in the middle of the region.

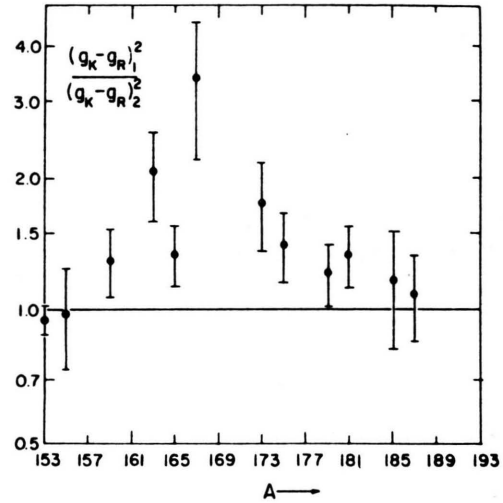


Fig. 1. The ratio of $(g_K - g_R)_1^2 / (g_K - g_R)_2^2$ as derived from Eq. 2. 1 refers to the transition $I+1 \rightarrow I$, and 2 refers to the transition $I+2 \rightarrow I+1$.

As the rotational $B(M1)$ values depend on the band parameters through the combination $(g_K - g_R)^2$, it is pertinent to examine whether these large discrepancies are due to a true incompatibility of the measured values with the rotational model, or whether they reflect only the hypersensitivity of $(g_K - g_R)^2$ to even small variations in the parameters if $g_R \sim g_K$.

⁶ Alpha-Beta and Gamma Ray Spectroscopy, ed. K. SIEGBAHN, North Holland Pub. Co., Amsterdam 1965.

⁷ M. MARTIN, P. MARMIER, and J. DE BOER, *Helv. Phys. Acta* **32**, 377 [1959].

⁸ E. M. BERNSTEIN and R. GRAETZER, *Phys. Rev.* **119**, 1321 [1960].

⁹ G. GOLDRING, H. M. LOEBENSTEIN, and R. BARLOUTAUD, *Phys. Rev.* **127**, 2151 [1962].

¹⁰ J. DE BOER, *Bull. Am. Phys. Soc.* **9**, (No. 1), 108 KC 11 [1964].

¹¹ N. D. S.

¹² G. GOLDRING and G. T. PAULISSEN, *Phys. Rev.* **103**, 1320 [1956].

In order to resolve this problem the parameters g_K , g_R were determined in the usual way, separately for the two transitions, from the $B(M1)$ values and the magnetic moment of the ground state, the rotational band value of which is given by:

$$\mu_I = \frac{I^2}{I+1} (g_K - g_R) + I g_R. \quad (3)$$

This evaluation of the parameters g_K , g_R is considered as a consistency check. If the parameters for the two transitions turn out to be substantially different, one cannot ascribe any immediate meaning

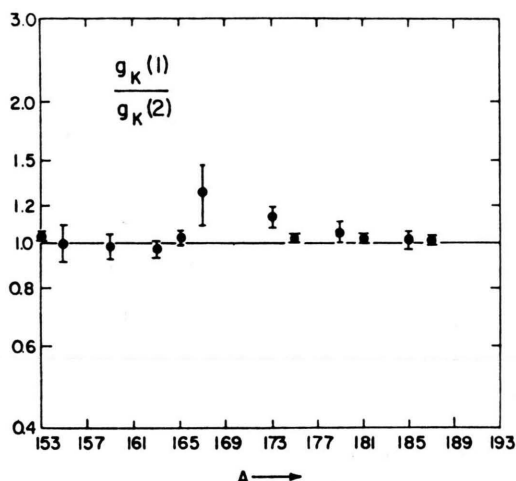


Fig. 2. The ratio of the parameter g_K for the two lowest transitions of the ground state rotational band in deformed Nuclei.

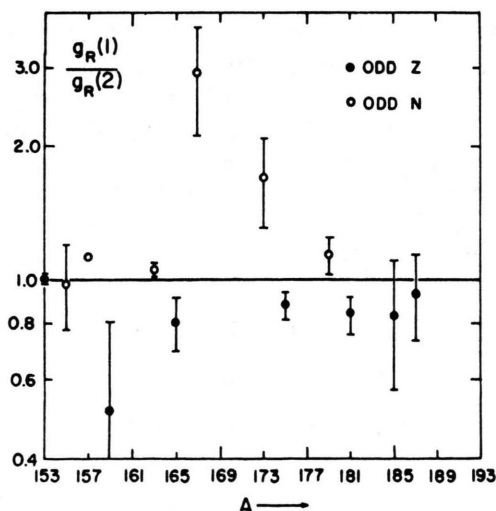


Fig. 3. The ratio of the parameter g_R for the two lowest transitions of the ground state rotational band in deformed Nuclei. In the case of Gd^{157} the error to the value could not be cited.

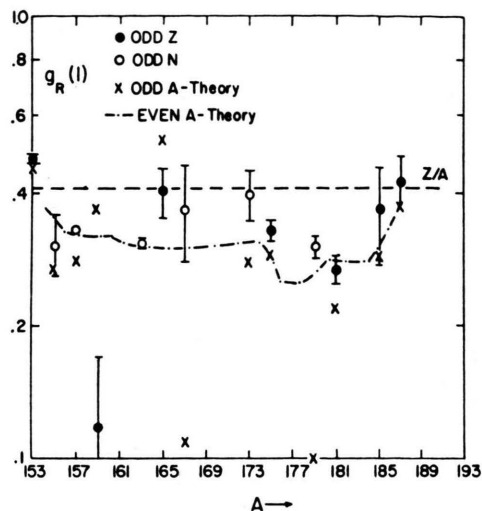


Fig. 4. The parameter g_R as derived from data on the transition $I+1 \rightarrow I$. The theoretical values are taken from Ref. ¹³. In the case of Gd^{157} the error to the value could not be cited.

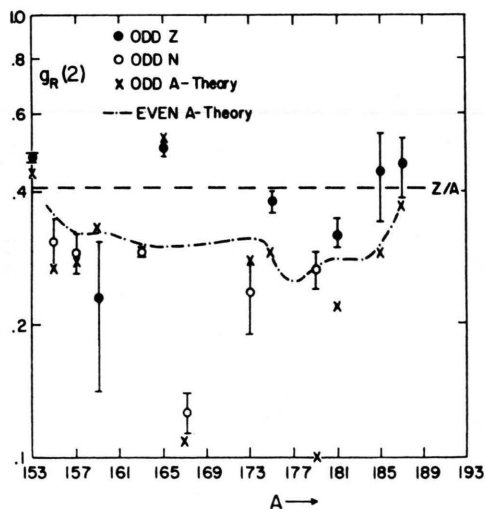


Fig. 5. The parameter g_R as derived from data on the transition $I+2 \rightarrow I+1$. The theoretical values are taken from Ref. ¹³.

to the individual parameters, but rather one has to conclude that in those cases the framework of the rotational motion represented by eqs. (1) – (3) is inadequate.

Figs. 2 and 3 show the ratios $g_K(1)/g_K(2)$ and $g_R(1)/g_R(2)$ respectively where 1 and 2 are the two transitions under consideration, and Figs. 4 and 5 show the values of g_R for each transition. The following conclusions can be drawn from these figures:

(i) The parameter g_K shows no variation at all in going from one transition to the other and it may properly be regarded as a constant of the band;

(ii) The parameter g_R does not in general assume the same value for the two transitions, and it is clearly these variations which give rise to the inconsistency in $B(M1)$ values commented on earlier (c. F. Fig. 1). The ratio $g_R(1)/g_R(2)$ is consistently smaller than unity for odd Z nuclei and larger than unity for odd N nuclei, the deviations being generally of 10–20 percent;

(iii) comparing the behavior of $g_R(1)$ and $g_R(2)$ as a function of A we find that $g_R(1)$ is almost constant and has a value of about 0.35, whereas $g_R(2)$ fluctuates quite appreciably. This would seem to indicate that the lower transition is better described by the rotational model than the upper one. We find however, surprisingly enough, that existing theoretical estimates of g_R^{13} , based on pair correlations, reproduce the fluctuating behavior of $g_R(2)$ much more faithfully than the smooth behavior of $g_R(1)$;

(iv) In two nuclei ^{167}Er , ^{173}Yb , both in the center of the rotational region, very large discrepancies are found between $g_R(1)$ and $g_R(2)$. The implied complete inadequacy of the parameter g_R is most unexpected and very difficult to fit in with all the other accumulated information in this region, all indicating pure collective rotational motion. It is

therefore important first of all to reexamine carefully the experimental evidence about those two nuclei. We shall deal with this question later on in this discussion.

Turning now to E2 transition probabilities we note that the ratio for the two transitions $I+2 \rightarrow I$; $I+1 \rightarrow I$ agrees quite well with Eq. (1)^{5, 12}, as may be seen from Fig. 6 which summarizes the relevant information from ref. 5. Fig. 7 summarizes in a similar way the ratios for the transitions: $I+2 \rightarrow I+1$; $I+1 \rightarrow I$. Here too the agreement with

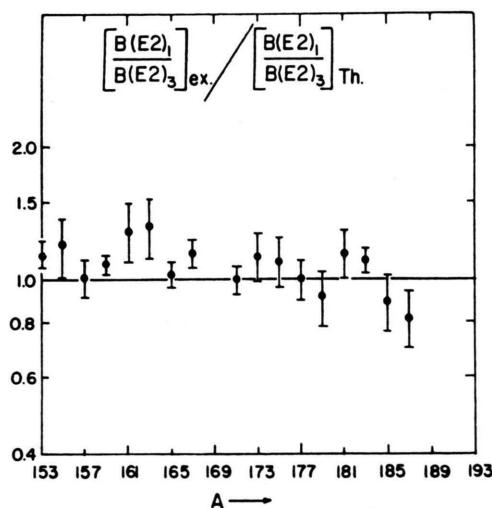


Fig. 6. Ratio of the E2 reduced transition probabilities for the transitions $I+1 \rightarrow I$ and $I+2 \rightarrow I$, divided by the respective theoretical value. The data are taken from Ref. 5.

1 Nucleus	2 I	3 $B(E2)_1$	4 $B(E2)_2$	5 $\left[\frac{B(E2)_1}{B(E2)_2} \right]_{\text{ex}} / \left[\frac{B(E2)_1}{B(E2)_2} \right]_{\text{Th}}$	6 $B(E2)_3$	7 $\left[\frac{B(E2)_1}{B(E2)_3} \right]_{\text{ex}} / \left[\frac{B(E2)_1}{B(E2)_3} \right]_{\text{Th}}$
^{153}Eu	5/2	1.74 ± 0.075	0.64 ± 0.12	2.3 ± 0.45	0.426 ± 0.024	1.14 ± 0.08
^{155}Gd	3/2	1.455 ± 0.067	1.07 ± 0.33	0.85 ± 0.26	0.507 ± 0.075	1.2 ± 0.18
^{157}Gd	3/2	1.495 ± 0.067	0.764 ± 0.12	1.22 ± 0.19	0.615 ± 0.05	1.01 ± 0.1
^{159}Tb	3/2	1.90 ± 0.053	1.18 ± 0.25	1.0 ± 0.21	0.735 ± 0.03	1.08 ± 0.05
^{163}Dy	5/2	1.92 ± 0.11	1.40 ± 0.23	1.16 ± 0.2	0.408 ± 0.06	1.32 ± 0.2
^{165}Ho	7/2	1.96 ± 0.056	1.55 ± 0.23	1.28 ± 0.19	0.427 ± 0.027	0.98 ± 0.07
^{167}Er	7/2	2.09 ± 0.064	1.13 ± 0.27	1.87 ± 0.45	0.407 ± 0.027	1.10 ± 0.08
^{173}Yb	5/2	2.18 ± 0.11	1.79 ± 0.46	1.06 ± 0.27	0.54 ± 0.06	1.13 ± 0.14
^{175}Lu	7/2	1.90 ± 0.05	1.47 ± 0.25	1.30 ± 0.22	0.39 ± 0.05	1.04 ± 0.13
^{179}Hf	9/2	1.465 ± 0.08	1.82 ± 0.44	0.91 ± 0.22	0.28 ± 0.036	0.91 ± 0.13
^{181}Ta	7/2	1.70 ± 0.12	1.09 ± 0.16	1.58 ± 0.26	0.33 ± 0.03	1.10 ± 0.13
^{185}Re	5/2	1.40 ± 0.14	0.97 ± 0.25	1.22 ± 0.34	0.44 ± 0.4	0.89 ± 0.13
^{187}Re	5/2	1.11 ± 0.11	0.81 ± 0.16	1.16 ± 0.26	0.38 ± 0.4	0.82 ± 0.12

Table 2. Properties of the electric quadrupole transitions of the ground state rotational band in odd A Nuclei with $K \neq \frac{1}{2}$ in the region $153 \leq A \leq 187$. Column 3: Values for the transition $I+1 \rightarrow I$, from Ref. 5. Column 4: Values for the transition $I+2 \rightarrow I+1$. Column 6: Values for the transition $I+2 \rightarrow I$ from Ref. 5.

¹³ Yu. T. GRIN and I. M. PAVLICHENKOV, J. Exp. Theor. Phys. USSR 14, 679 [1962].

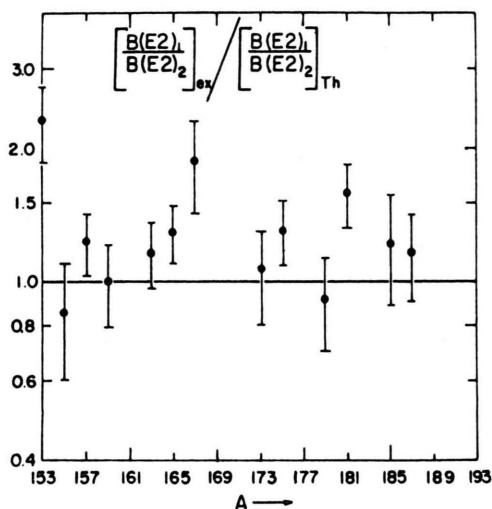


Fig. 7. Ratio of the E2 reduced transition probabilities for the transitions $I+1 \rightarrow I$ and $I+2 \rightarrow I+1$, divided by the respective theoretical value.

the rotational model is in general quite good. If in particular one computes Q_0 from Eq. (1) one finds that, with three exceptions, the values of Q_0 for the transitions:

$$I+1 \rightarrow I, I+2 \rightarrow I, I+2 \rightarrow I+1$$

all agree to within ten percent. The exceptions are: ^{153}Eu , ^{167}Er and ^{181}Ta . In view of the good agreement with the rotational value of the ratio for the $I+1 \rightarrow I$; $I+2 \rightarrow I$ transitions, we conclude that in all three cases the $B(E2)$ value for the $I+2 \rightarrow I+1$ transition is substantially smaller than the rotational value. This conclusion is well substantiated for ^{153}Eu and ^{181}Ta , situated respectively at the beginning and close to the end of the region of strong deformation as the experimental evidence is derived from a number of measurements of different types (branching ratio, lifetimes, angular distributions and conversion coefficient ratios) all of which are internally consistent.

^{167}Er is unique in that both the E2 and M1 transition probabilities deviate strongly from the rotational motion values. If one suspects an experimental error as the source of these inconsistencies, one finds that of all the types of measurements performed in this case, only one — the branching ratio of gamma rays from the second excited level — could (if erroneous) be responsible for both discrepancies.

If the value of the branching ratio is considered reasonably well established and with the measurements of Ref. 5 the quantities $B(E2; I+1 \rightarrow I)$, $B(E2; I+2 \rightarrow I)$ and $B(M1; I+2 \rightarrow I)$ can be considered as reliably determined. The quantities $B(E2; I+2 \rightarrow I+1)$ and $B(M1; I+1 \rightarrow I)$ are now determined from angular distribution measurements of the gammas of the $I+2 \rightarrow I+1$ and $I+1 \rightarrow I$ transitions respectively^{3,7}. All of these measurements have to be wrong if the E2 and M1 anomalies are both to be blamed on experimental errors.

We note also that similar anomalies have been found in a number of odd A nuclei and in particular in Er^{167} in higher transitions¹⁴.

The review of DE BOER and ROGERS concerned itself mainly with the parameter g_K , and we find now that the value of this parameter and the conclusions drawn by DE BOER and ROGERS are not modified by more recent experimental work. However, as far as the parameter g_R is concerned we conclude now that it is not a "good" parameter in the region considered. It exhibits appreciable, and in ^{167}Er and ^{173}Yb quite large variations if derived independently from two different transitions. As a consequence M1 transition probabilities are definitely less faithfully represented by the rotational motion theory than, in most cases, E2 transitions.

Acknowledgements

The authors wish to thank Mr. MARK GOLDBERG for his help in the calibration of the Germanium Counter.

Professor GENTNER has played a decisive role in the creation of the Heinemann Laboratory in Rehovoth. On the occasion of his sixtieth anniversary, in recognition and gratitude for his generous efforts and critical guidance we find it appropriate and proper to present this note as a summary and conclusion of work carried out at our Laboratory in recent years and of some of the first experiments on the tandem accelerator at the Heinemann Laboratory.

Note added in proof: The branching ratio λ for cross-over to cascade gamma radiation from the $\frac{1}{2}^+$ level of ^{167}Er has recently been measured with a $\text{Ge}(\text{Li})$ counter as: $\lambda = 0.30 \pm 0.03$ (F. BOEHM et al. — private communication). With this value of λ the discrepancies in the transition probabilities are appreciably reduced. A large discrepancy remains however in the $B(M1)$ values of the two transitions. A similar measurement recently carried out by the authors of the present communication yielded the value:

$$\lambda = 0.35 \pm 0.05.$$

¹⁴ J. DE BOER and G. D. SYMONS, Congrès International de Physique Nucléaire, Paris 1964.